BMath Algebra-IV
Back paper-2019

Instructions : Total time-3 hours. **Solve any five problems**. You may use results done in the class, without proofs. If you wish to use any assignment/tutorial problem, you should incluse its solution in your answer.

1. Let L/k be a finite Galois extension with Galois group G. Let H be a subgroup of G. Prove that there exists $\alpha \in L$ such that the stabilizer subgroup of α in G equals H. (10)

2. Let L/k be a finite Galois extension and $f(X) \in L[X]$. Prove that there exists a nonzero polynomial $g(X) \in L[X]$ such that $f(X)g(X) \in k[X]$. (10)

3. Let k be a field and $f(X) \in k[X]$ be a nonconstant polynomial of degree n having distinct roots. Let $K = Split(f(X), k)$ and $G := Gal(K/k)$. Prove that if the number of orbits of G on the set of roots of $f(X)$ in K is r, then $f(X) = f_1(X) \cdots f_r(X)$ for some irreducible polynomials $f_i(X) \in k[X]$. (10)

4. Let L/k be a Galois extension and suppose $\sigma \in G := Gal(L/k)$ has order 2. Prove that L^σ, the fixed field of σ, is Galois over k if and only if σ belongs to the centre of G. (10)

5. Let p be a prime and ζ_p be a primitive pth root of unity in $\mathbb{C}$ and let $L = \mathbb{Q}(\zeta_p)$. Prove that for every divisor d of $p-1$, there exists a subfield $M \subset L$ such that $[M : Q] = d$. (10)

6. Construct a Galois extension L of $\mathbb{Q}$ with $Gal(L/\mathbb{Q}) \cong \mathbb{Z}/8\mathbb{Z}$. (10)